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TRANSCRIPT OF MATHEMATICS COURSEWORK

Harvard University

Math 60R – Honors Senior Thesis, Melanie Wood and Ashvin Swaminathan, Fall 2024. Honors senior thesis project on 3-torsion in class groups of orders in global fields. The goal of the project is to compute the average size of the 3-torsion subgroup of the class group of an order in a global field via the recently developed geometry-of-numbers methods over global fields developed by Bhargava, Shankar, and Wang. Grade: SAT

Math 223aR – Introduction to the Theory and Computational Practice of Elliptic Curves, Noam Elkies, Fall 2024.

An introduction to elliptic curves à la Silverman's text. Topics include: algebraic varieties; algebraic curves; the geometry of elliptic curves; formal groups of elliptic curves; elliptic curves over finite fields, the complex numbers, local fields, and global fields; the Mordell–Weil and Lutz–Nagell theorems. Final Project: the average size of the 3-isogeny Selmer groups of elliptic curves. *Textbook: The Arithmetic of Elliptic Curves* by Joseph Silverman Grade:

Math 299R – Arithmetic and Exposition, Melanie Wood, Spring 2024.

Continuation of research project with Melanie Wood studying the distribution of cokernels of Hermitian matrices with independent random entries taking values in the rings of integers of quadratic fields with their Hermitian pairings. Final Project: talk at the arithmetic statistics seminar. Grade: A

Math 223b - Algebraic Number Theory, Salim Tayou, Spring 2024.

A continuation of Math 223a. Topics include: global class field theory, group cohomology, Brauer groups and central simple algebras, analytic methods in algebraic number theory, Tate's thesis, and a brief introduction to the Langlands program. Final Project: Explicit class field theory for elliptic curves with complex multiplication.

Textbook: Algebraic Number Theory by Jürgen Neukirch; *Algebraic Number Theory* by J. W. S. Cassels and Albrecht Fröhlich; and Chapter II of *Advanced Topics in the Arithmetic of Elliptic Curves* by Joseph Silverman

Grade: A

Math 222 – Lie Groups and Lie Algebras, Noam Elkies, Spring 2024. The theory of Lie groups and Lie algebras, including the classification of semisimple Lie algebras. Final Project: the Peter–Weyl theorem and its applications. *Textbook: Representation Theory: A First Course* by Fulton and Harris. Grade: A

Math 91R – Supervised Reading and Research, Melanie Wood, Fall 2023.

Research project with Melanie Wood studying the distribution of cokernels of Hermitian matrices with independent random entries taking values in the rings of integers of quadratic fields with their Hermitian pairings.

Grade: A

Phil 248R – Equality, Barry Mazur, Eric Maskin, Amartya Sen, Fall 2023. Seminar course on the concept of equality from a philosophical, mathematical, economic, and legal perspective. Final paper topic: Isomorphism. Grade: A

Math 223a – Algebraic Number Theory, Melanie Wood, Fall 2023.

Course on local fields. Topics include: local fields, via completions and inverse limits, and their topology; multiplicative structure of local fields, Hensel's lemma, unramified extensions of local fields; tame absolute Galois group of local fields; inertia groups, different, discriminant, Artin conductors; group cohomology; local class field theory.

Textbook: Local Fields by Jean-Pierre Serre and *Algebraic Number Theory* by Jürgen Neukirch Grade: A

Math 213a – Advanced Complex Analysis, Curtis McMullen, Fall 2023.

Second course on complex analysis on the plane, sphere and complex tori. Topics include: basic complex analysis; Weierstrass products; Mittag-Leffler theorem; trigonometric functions; the Gamma function; Riemann mapping theorem; Schwarz-Christoffel formula; Bloch's theorem; Picard's theorem; and elliptic functions.

Textbook: Complex Analysis by Lars Ahlfors and *Conformal Mapping* by Zeev Nehari. Grade: A

Math 91R – Supervised Reading and Research, Melanie Wood, Spring 2023.

Continuation of research project with Melanie Wood studying the distribution of sandpile groups of Erdős-Rényi random graphs with their pairings.

Grade: A

Math 213b – Riemann Surfaces, Peter Kronheimer, Spring 2023.

An introduction to the theory of Riemann surfaces. Topics include: branched coverings; analytic curves; Riemann-Hurwitz; de Rham and Dolbeault cohomology; classification of genus-1 Riemann surfaces; divisors; Jacobians; Abel's theorem and the Abel-Jacobi theorem; Riemann-Roch; and the uniformization theorem.

Textbook: Riemann Surfaces by Simon Donaldson. Grade: A

Math 280Y – Arithmetic Statistics, Fabian Gundlach, Spring 2023.

Introduction to the parameterize-and-count method due to Bhargava and his collaborators. Topics include: random primes; random polynomials; counting local fields; counting number fields (Malle's conjecture); random number fields (Cohen-Lenstra heuristics); sieves, lattice-point and orbit-counting methods; and heuristics.

Grade: A

Math 91R – Supervised Reading and Research, Melanie Wood, Fall 2022.

Research project with Melanie Wood studying the distribution of Sandpile groups (and their pairings) of a random graph generated with respect to the Erdős-Rényi model. Grade: A

Math 232a – Introduction to Algebraic Geometry I, Dori Bejleri, Fall 2022.

Graduate-level treatment of algebraic geometry. Topics include: category theory; affine schemes; general schemes; morphisms of schemes; varieties; quasi-coherent sheaves; dimension; line bundles and divisors. *Textbook: The Rising Sea: Foundations of Algebraic Geometry* by Ravi Vakil. Grade: A

Math 231a – Algebraic Topology, Andrew Senger, Fall 2022.

Graduate-level introduction to algebraic topology: Topics covered include singular homology and its basic properties; CW complexes and cellular homology; the universal coefficient and Künneth theorems; cohomology and the cup product; and Poincaré duality. *Textbook: Algebraic Topology* by Allen Hatcher. Grade: A

Math 129 – Number Fields, Barry Mazur, Spring 2022.

Algebraic number theory: number fields; unique factorization of ideals; finiteness of class group; structure of unit group; Frobenius elements; local fields; ramification; weak approximation; adeles, and ideles. *Textbook: Number Fields* by Daniel Marcus. Grade: A

Math 137 – Algebraic Geometry, Fabian Gundlach, Spring 2022.

Algebraic geometry: affine and projective varieties; Hilbert's Nullstellensatz; Gröbner bases; dimension theory; tangent spaces; curves; intersection numbers; Bézout's theorem; divisors and linear systems; and the Riemann–Roch theorem.

Textbook: Algebraic Curves by Fulton. Grade: A

$Freshman\ Seminar\ 51E- \ The\ Story\ of\ the\ Alternating\ Sign\ Matrix\ Conjecture,$

Lauren Williams, Spring 2022.

Freshman seminar illustrating how research in mathematics actually progresses, using recent examples from the field of algebraic combinatorics. Specifically, a survey of the search for and discovery of a proof of a formula conjectured by Mills-Robbins-Rumsey in the early 1980's: the number of n by n alternating sign matrices.

Textbook: Proofs and Confirmations: The Story of the Alternating Sign Matrix Conjecture by Bressoud. Grade: SAT

Math 55b – Studies in Real and Complex Analysis, Denis Auroux, Spring 2022.

Rigorous introduction to real and complex analysis and topology. Topology topics include: basic point-set topology up to Urysohn's Metrization Theorem and basic algebraic topology from homotopy up to Seifert van Kempen. Real analysis topics include: basic real analysis in single and multiple variables up to a treatment of differential forms and Stokes' Theorem. Complex analysis topics include: holomorphic and meromorphic functions; the Riemann sphere; Cauchy's Theorem and bound; Taylor and Laurent serie; the maximum, open-mapping, and argument principles; residue calculus and keyhole integration; and sum and product expansions.

Textbook: Topology by Munkres; *Principles of Mathematical Analysis* by Rudin; *Complex Analysis* by Ahlfors.

Grade: A

Stat 110 – Introduction to Probability, Joseph Blitzstein, Fall 2021.

Topics covered: counting; basic probability; conditional probability; random variables; expected value; conditional expectation; important discrete and continuous distributions; jointly distributed random variables; law of large numbers and the central limit theorem; and Markov chains. *Textbook: Introduction to Probability* by Joseph Blitzstein and Jessica Hwang. Grade: A

Math 55a – Studies in Linear Algebra and Group Theory, Denis Auroux, Fall 2021.

A rigorous introduction to group theory and linear algebra. Group theory topics include basic group theory up to the Sylow theorems and the classification of finitely generated abelian groups. Linear algebra topics include basic linear algebra up to Jordan normal form and tensor products/multilinear algebra. Some category theory and an introduction to representation theory, namely the representation theory of finite

groups; representations of S_3 ; group characters and character tables; and induced representations and Frobenius reciprocity. *Textbook: Algebra* by Artin; *Linear Algebra Done Right* by Axler; *Representation Theory: A First Course* by Fulton and Harris. Grade: A

University of Colorado at Boulder

Math 4900 – Independent Study, Sebastian Casalaina-Martin, Summer 2021. Introduction to point-set topology (continuity, compactness, and connectedness); bilinear forms and the spectral theorem; the linear groups and Lie algebras; representations of finite groups and characters; field theory; Galois theory; and further topics in algebra. *Textbook: Algebra* by Artin Chapters 8, 9, 10, 11.8-9, 15, 16; *Algebra* by Lang, Chapters 5 and 6;

Mathematical Analysis by Browder, Chapter 6. Grade: A

Math 4140 – Abstract Algebra II: Algebras and Rep. Theory, Tianyuan Xu, Spring 2021. Representation theory of algebras, with an emphasis on group algebras, path algebras of quivers, and their representations. Topics: composition series; the Jordan-Hölder Theorem; semisimple algebras; Jacobson radicals; the Artin-Wedderburn Theorem; Maschke's Theorem; the Krull-Schmidt Theorem; group characters.

Textbook: Algebras and Representation Theory by Erdmann and Holm. Grade: A

Math 4001 – Analysis II, Independent Study, Judith Packer, Spring 2021.

Infinite series of real numbers; convergence and uniform convergence of sequences and series of functions; Taylor's Theorem; analysis in multiple variables: differentiation, the chain rule, mean value theorem, inverse mapping theorem and implicit function theorem. Undertook an independent project in which I studied and then presented on integration of multivariable functions, differential forms, integration of differential forms and the generalized Stokes' Theorem.

Textbook: Undergraduate Analysis by Lang, Chapters IX, XV, XVII; *Principles of Mathematical Analysis* by *Rudin*, Chapters 3, 7, 9; *Calculus on Manifolds* by Spivak, Chapters 1-4. Grade: A

Stanford University-Level Online

XM609 – Modern Algebra, Margarita Kanarsky, Fall 2020.

Topics: groups; quotient groups; symmetry groups in three dimensions; Pólya-Burnside method; rings and fields; polynomial and Euclidean rings; quotient rings; field extensions; and geometrical constructions. *Textbook: Modern Algebra* with Applications by Gilbert. Grade: A

XM615 – Real Analysis, Tina Lal, Fall 2020.

Topics: sequences; series; limits; continuity and uniform continuity; sequences and series of function; uniform convergence; differentiation.

Textbook: Elementary Analysis: The Theory of Calculus by Ross. Grade: A

XM452 – Elementary Theory of Numbers, Margarita Kanarsky, Summer 2020.

Topics: Euclid's algorithm; divisibility; prime numbers; congruence of numbers; theorems of Fermat, Euler, Wilson; Lagrange's theorem; residues of power, quadratic residues; magic squares; continued fractions; Diophantine equations; quadratic fields and quadratic integers. *Textbook: An Introduction to Number Theory* by Stark.

Grade: A

XM511 – Linear Algebra, Margarita Kanarsky, Spring 2020.
Topics: matrices; linear equations; vector spaces; bases; coordinates; linear transformations; eigenvectors; eigenvalues; and diagonalization.
Textbook: Linear Algebra: An Introduction by Bronson and Costa.
Grade: A

Kent Denver School

Honors Multivariable Calculus, Cheryl Askay, Fall 2019 and Spring 2020. *Textbook: Calculus* by Larson. Grade: A+

18.03 – MIT OCW Differential Equations Independent Study, Fall 2020.

Completed independent study of a Differential Equations course (18.03) from MIT's Opencourseware. Topics include: solution of first-order ODE's; linear ODE's, especially second order with constant coefficients; undetermined coefficients; sinusoidal and exponential signals: oscillations, damping, resonance; complex numbers and exponentials; Fourier series, periodic solutions; delta functions, convolution, and Laplace transform methods; and matrix and first-order linear systems: eigenvalues and eigenvectors.

Textbook: Elementary Differential Equations with Boundary Value Problems by Edwards and Penney. Grade: P